## 2016 James S. Rickards Fall Invitational

For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

- 1. What is the value of the geometric series  $i + i^2 + i^3 + i^4 + \ldots + i^{2016}$ ? (Note:  $i = \sqrt{-1}$ ) (A) -1 (B) -i (C) -1 + i (D) 1 + i (E) NOTA
- 2. Mario trips and hits his head on a massive brick. A question pops out. What is the sum of the real roots of  $f(x) = x^4 + x^3 8x^2 + 4x 48$ ? (A) -4 (B) -6 (C) -1 (D) -7 (E) NOTA
- 3. Accountant Peter Burke comes across a difficult question during his day at work. Perhaps you can help him. How many zeroes are at the end of 681!?
  (A) 169
  (B) 168
  (C) 167
  (D) 166
  (E) NOTA
- 4. Find the area of the locus of points (x, y) in the xy-plane such that  $|x| + |y| \le 8$ . (A) 64 (B) 256 (C) 128 (D) 32 (E) NOTA

5. Edward Elric is tasked with finding the real roots of the following:  $f(x) = e^{2x} + 3e^x - 3^3 \cdot 2$ . What are the real roots? (A)  $\ln -9$ ,  $\ln 6$  (B)  $\ln -9$  (C)  $\ln -6$ ,  $\ln 9$  (D)  $\ln 6$  (E) NOTA

- 6. Find the number of integers x that satisfy |x+3| > |2x-4|. (A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA
- 7. Deadpool often makes fun of summations. As a result he is punished by the evil overlords and given a double sum to solve. Evaluate the double sum:

(A) 
$$\frac{1}{2}$$
 (B)  $\pi$  (C)  $\infty$  (D) 1 (E) NOTA

- 8. Batman is determined to pull off a mission. He needs the code to a safe which can be found by solving the following: 36 67 845
  - $\begin{vmatrix} 0 & 55 & 98 \\ 0 & 0 & 23 \end{vmatrix}$ . What is the code? (A) 45540 (B) 54540 (C) 45440 (D) 54450 (E) NOTA
- 9. Humpty Dumpty must bypass a massive stone wall. He can do so with the answer to the next question. Which of the following is an asymptote of the graph of  $25x^2 9y^2 100x + 54y = 206$ ? (A) -5x - 3y = 19 (B) 5x + 3y = 19 (C) 5x + 3y = 1 (D) 5x - 3y = -19 (E) NOTA
- The Axe Man cuts down trees for a living. The number of trees he must cut down every day can be found by solving the following:
   (log. 49)<sup>2</sup>

$$\alpha^{(\log_4 49)}$$
If  $\alpha^{(\log_4 49)} = 16$ , how many trees must the Axe Man chop down?  
(A) 256 (B) 2401 (C) 32 (D) 16 (E) NOTA

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## Algebra 2 Individual

11. Find all values of x such that  $\frac{x^2 - 3x}{x^2 - 10x + 24} \ge 0.$  Express the answer in interval notation. (A)  $(\infty, 0] \cup [3, 4) \cup (6, \infty)$  (B)  $(-\infty, 4) \cup (6, \infty)$  (C)  $(-\infty, 0] \cup [3, 4) \cup (6, \infty)$  (D)  $(-\infty, \infty)$  (E) NOTA

12. Find the minimum distance between the line  $\frac{6}{13} - \frac{4}{13}x = y$  and the point (3,7).

(A) 
$$\frac{97\sqrt{7}}{135}$$
 (B)  $\frac{97}{35}$  (C)  $97\sqrt{175}$  (D)  $\frac{97\sqrt{185}}{185}$  (E) NOTA

13. What is  $\sqrt{12 + \sqrt{12 + \sqrt{12 + \dots}}} + \frac{1}{2 + \frac{3}{4 + \frac{1}{2 + \frac{3}{\dots}}}}$ (A)  $\frac{3 + \sqrt{33}}{2}$  (B) 5 (C)  $3 + \sqrt{2}$  (D)  $\frac{3 \pm \sqrt{33}}{2}$  (E) NOTA

14. Approximate the geometric series  $1 + \frac{23}{50} + \left(\frac{23}{50}\right)^2 + \ldots + \left(\frac{23}{50}\right)^{2016}$  to the nearest thousandth. (A) 0.460 (B) 1.852 (C) 2.016 (D) It cannot be solved without a calculator. (E) NOTA

- 15. Find the slope of the line that is tangent to  $y = 2x^2 + 4x$  at x = -1. (A) 0 (B) 3 (C) 6 (D) 9 (E) NOTA
- 16. Find the coordinates of the center of the circle with equation  $\alpha\beta + (-6 7i)\alpha + (-6 + 7i)\beta = 0$ , where  $\alpha = x + yi$ and  $\beta$  is the complex conjugate of  $\alpha$ . (A) (6,7) (B) (-6,7) (C) (6,-7) (D) (-6,-7) (E) NOTA

17. If z = x + yi, which of the following best describes the graph of |z + 13i - 7| = 4? (A) Hyperbola (B) Circle (C) 2 intersecting lines (D) Ellipse (E) NOTA

18. Evaluate the infinite arithmetico-geometric series:  $\frac{1}{6} + \frac{1}{18} + \frac{1}{72} + \frac{1}{324} + \frac{5}{7776} + \dots$ (A)  $\frac{1}{4}$  (B) 1 (C)  $\frac{4}{21}$  (D)  $\frac{6}{25}$  (E) NOTA

19. If  $\frac{3}{3+\sqrt{2}+\sqrt{11}}$  can be expressed in the form  $\frac{a\sqrt{2}-b-c\sqrt{22}}{4}$ , find the value of a+b+c. (A) 4 (B) 12 (C) 6 (D) 2 (E) NOTA

20. How many asymptotes does the following graph have?

(A)

$$\frac{x^3 - 12x^2 + 47x - 60}{x^2 - 7x + 12}$$
1 (B) 2 (C) 3 (D) 0 (E) NOTA

21. If  $x^3 - y^3 = 180$  and x - y = 18, what is  $x^2 + 4xy + y^2$ ? (A) 10 (B) -344 (C) -304 (D) 300 (E) NOTA 22. How many of the following statements are true?

I. When graphed on the Argand plane, 3Re(z) + 4Im(z) = 17.5, is a line (Note: z is a complex number).

II. In the Argand plane, real numbers are represented on the vertical axis.

III. The domain of  $\log_3 x$  is  $x \ge 0$ .

IV. The function  $5(y-1) = (x-3)^2$  is one-to-one.

(A) 2 (B) 1 (C) 4 (D) 3 (E) NOTA

23. Nihar and Neal are arguing over how many roots  $x^4 = 1$  has. Nihar says it has 5 roots and Neal says it has 6 roots. How many roots does  $x^4 = 1$  have? (A) 4 (B) 3 (C) 2 (D) (E) NOTA

24. Karthik the Giant's height is equal to the minimum of the function  $3x^2 + 4x + 2$ . What is Karthik's height? (Assume that in his odd world, there are no measurements for height – it is merely a number.)

- (A)  $\frac{2}{3}$  (B) 1 (C)  $\frac{4}{3}$  (D)  $\frac{5}{3}$  (E) NOTA
- 25. What is the sum of the squares of the roots of  $x^2 9x + 11$ ? (A) -113 (B) 81 (C) 59 (D) 65 (E) NOTA
- 26. If  $x + \frac{1}{x} = 23$ , what is  $x^4 + \frac{1}{x^4}$ ? (A) 279839 (B) 277729 (C) 279841 (D) 277727 (E) NOTA
- 27. If  $f(x) = x^2 + 16x + 67$ , what is  $f^{-1}(3)$ ? (A) -10 (B) -9 (C) -8 (D) -7 (E) NOTA
- 28. What is the coefficient of the third term in the expansion of  $(x y)\overline{3}$ ? (A)  $-\frac{1}{9}$  (B)  $\frac{1}{3}$  (C)  $\frac{2}{3}$  (D) 1 (E) NOTA
- 29. What is the remainder when  $x^5 6x^4 + 3x^2 7 + 4x$  is divided by (x 2)? (A) -78 (B) -45 (C) 51 (D) 40 (E) NOTA
- 30. Imagine you are at a Mu Alpha Theta competition. You enter a testing room in which there are three desks and one proctor. This room and competition is quite different from what you are used to. The proctor tells you that each desk has a ticket. This ticket will automatically dictate your score for this competition. On two of the desks, the ticket says "SCORE: -30". On one desk the ticket reads "SCORE: 120." You want to receive a score of 120. You choose a desk. The proctor goes to another desk and shows you the ticket that was on it. The ticket says "SCORE: -30". The proctor now gives you an option to switch desks, or remain in your current one. If you switch, what is the probability that you receive a perfect score of 120 for this competition (the tickets are covered up until you have reached your decision and the proctor unveils them)?
  - (A)  $\frac{1}{3}$  (B)  $\frac{1}{2}$  (C)  $\frac{2}{3}$  (D)  $\frac{1}{4}$  (E) NOTA